A total neutrino conversion in the Earth without a resonance.

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Abstract

The neutrino oscillation enhancement in the Earth-type medium *mantle – core – mantle* is discussed. It is noted that the total conversion is possible both for a resonant matter density and a nonresonant one. A useful parameterization, for the representation of the transition probability for neutrinos and antineutrinos in a single plot, is proposed.

The matter effect of the Earth on neutrino oscillations, in the interesting region of oscillation parameters for the solar and atmospheric neutrinos, is widely discussed now [1, 2, 3, 4, 5, 6, 7]. Due to the specific multilayer structure of the Earth, a new effect [6, 7] of an enhancement of the oscillations of massive neutrinos is possible. In contrast to MSW *resonance* effect [8] this new effect occurs due to a maximal *constructive interference* among transition amplitudes, which give contribution to the total amplitude in the multilayer case.

A good approximation for the Earth interior is a two-layer model with two basic structures: the mantle and the core. These structures have slowly increasing densities from the surface of the Earth to its center with a sharp leap on their border. Therefore, we can consider the mantle and the core densities on the neutrino trajectories as different constants. This assumption

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leads to simple analytical formulae for the neutrino transition probability (see, for example, [2]).

In the case of oscillations between two ultra-relativistic neutrino species in vacuum, there are two parameters: the vacuum mixing angle ϑ_0 and the ratio between the neutrino squared mass difference and the neutrino energy $\Delta m^2/E$. We assume for definiteness that $\Delta m^2 > 0$ and

$$0 < \vartheta_0 \leq \pi/4, \tag{1}$$

i.e. $\cos(2\vartheta_0) > 0$. If the vacuum mixing angle ϑ_0 is small, the neutrino transition probability is suppressed. Therefore, in this case the experimental search of oscillations is extremely difficult.

On the other hand, a medium can effect oscillations and, in particular, enhance them. For oscillations in a medium, a difference $V_{\alpha\beta}$ ($\alpha \neq \beta = e, \mu, \tau, s$) between the effective potentials of different neutrino species ν_α and ν_β can arise. For neutrinos it can be either positive

$$V_{e\mu} = \sqrt{2} G_F N_e > 0, \tag{2}$$

or negative

$$V_{\mu s} = -\sqrt{2} G_F N_n/2 < 0, \tag{3}$$

where N_e and N_n are the electron and neutron number densities of the medium. For antineutrinos $V_{\alpha\beta}$ is replaced by

$$V_{\bar{\alpha}\bar{\beta}} = -V_{\alpha\beta}. \tag{4}$$

The matter mixing angle ϑ is given by the well-known expression

$$\cos(2\vartheta) = \frac{1}{\Delta E}\left(\frac{\Delta m^2}{2E}\cos(2\vartheta_0) - V_{\alpha\beta}\right), \tag{5}$$

where

$$\Delta E = \frac{\Delta m^2}{2E}\sqrt{\left(\cos(2\vartheta_0) - \frac{2EV_{\alpha\beta}}{\Delta m^2}\right)^2 + \sin^2(2\vartheta_0)} \tag{6}$$

being the difference between the energies of the two neutrino energy-eigenstates in the medium.

In order to present a comparison of the probabilities of neutrinos and antineutrinos in a single figure, we extend formally the range of the vacuum mixing angles (1) to the region

$$0 < \vartheta_0 < \pi/2 \tag{7}$$

in such a way that $\cos(2\vartheta_0) < 0$ corresponds to the antineutrino case, keeping the same $V_{\alpha\beta}$ for neutrinos and antineutrinos. This is possible, because in the two-species case we can obtain antineutrino evolution equation from the neutrino one

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \frac{\Delta E}{2}\begin{pmatrix} -\cos(2\vartheta) & \sin(2\vartheta) \\ \sin(2\vartheta) & \cos(2\vartheta) \end{pmatrix}\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} \tag{8}$$

by the formal substitutions: $\cos(2\vartheta_0) \to -\cos(2\vartheta_0)$ and $\nu_\alpha \to \bar{\nu}_\beta$, $\nu_\beta \to \bar{\nu}_\alpha$. As far as $P_{\alpha\beta(\bar{\alpha}\bar{\beta})} = P_{\beta\alpha(\bar{\beta}\bar{\alpha})}$, we can plot the *continuous* total transition probability $P_{\alpha\beta}$ for neutrinos and antineutrinos in a single figure, using extended region (7) for the vacuum mixing angles.

When the MSW resonance condition

$$\frac{\Delta m^2}{2E}\cos(2\vartheta_0) = V_{\alpha\beta} \tag{9}$$

is fulfilled, the matter mixing angle can be maximal, $\vartheta = \pi/4$, even in the case of a small vacuum mixing angle ϑ_0. In this case the neutrino transition probability can reach its maximal value $P_{\alpha\beta} = 1$. It can be realized either for neutrinos or for antineutrinos. In a constant density *homogeneous* medium the maxima of neutrino transition probability lie on the curve (9) in $(\cos(2\vartheta_0), \Delta m^2/E)-$plane. The positions of the maxima depend on the distance X, travelled by the neutrinos or antineutrinos, and are defined by the phase condition

$$\phi = \Delta E X = (2k+1)\pi, \qquad k = 0, 1, 2, \dots\ . \tag{10}$$

When the (anti)neutrinos arrive to the detector at nadir angle greater than 33°, they pass only through the Earth mantle, which is assumed to have a constant density $\rho_m \cong 4.5$ g/cm^3. Therefore, we can consider it as a simple case of a neutrino propagation in a constant density homogeneous medium. The contours of the analytically calculated transition probability,

at the nadir angle $h = 70^\circ$ for different oscillation parameters in the cases of $\overset{(-)}{\nu}_\mu \leftrightarrow \overset{(-)}{\nu}_\tau$ and $\overset{(-)}{\nu}_\mu \leftrightarrow \overset{(-)}{\nu}_s$ oscillations, are shown in Fig. 1. In the case of $\overset{(-)}{\nu}_\mu \leftrightarrow \overset{(-)}{\nu}_\tau$ oscillations, $V_{\mu\tau} = 0$. This corresponds to a vacuum case, when a total conversion $P_{\mu\tau} = 1$ takes place only at a maximal vacuum mixing angle $\vartheta_0 = \pi/4$, i.e. $\cos(2\vartheta_0) = 0$ (Fig. 1a). The transition probability has a symmetrical form and there is no difference between neutrino and antineutrino cases. The later case of $\overset{(-)}{\nu}_\mu \leftrightarrow \overset{(-)}{\nu}_s$ oscillations allows a total resonance conversion for antineutrino oscillations $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_s$, and a supression of the transition probability for $\nu_\mu \leftrightarrow \nu_s$ case (Fig. 1b). This feature enables us to distinguish between these cases for the atmospheric neutrinos. In the following we consider just the latter case which is effected by a matter distribution.

In the case of smaller nadir angles $h \leq 33^\circ$, (anti)neutrinos pass also through the Earth core, which density we assume to be constant $\rho_c \cong 11.5$ g/cm^3. This leads to simple analytical expressions for the neutrino transition probability, which has been analyzed in [7]. Due to the strong interference between the amplitudes in the mantle and the core, a total conversion *even* for neutrinos $P_{\mu s} = 1$ can occur. At the nadir angles just below 33° the absolute maxima move away from the curve (9) and their interpretation in the terms of the MSW resonances becames meaningless. As it was shown in [7], in the three-layer case of the Earth profile, the total (anti)neutrino conversion is possible in the infinite two-dimensional region of the oscillation parameters

$$\textit{region } \mathcal{A}: \quad \begin{cases} \cos(2\vartheta_c) \leq 0 \\ \cos(2\vartheta_c - 4\vartheta_m) \geq 0, \end{cases} \tag{11}$$

where ϑ_m and ϑ_c are the mixing angles in the mantle and the core, correspondingly. For fixed Δm^2 and the vacuum mixing angle ϑ_0, the conditions (11) give the allowed values of neutrino energy E, at which a total conversion in the Earth is possible. In contrast to the MSW resonance condition (9), where only single value of E is possible, there exists a continuum of different solutions for E. Moreover, the region $\mathcal{A}$ is wider than the analogous region in the two-layer case considered in [6]. For $\rho_c > 2\rho_m$, which has place for the Earth, the region $\mathcal{A}$ overlaps the parameter space $\cos(2\vartheta_0) > 0$, where the MSW resonance condition cannot be satisfied, due to the different signs on the left and on the right hand side of eq. (9). However, a total neutrino conversion is possible. It is somewhat opposite to the common opinion that in

this case the matter suppresses oscillations and the total neutrino conversion cannot occur.

The positions of the absolute maxima in the region $\mathcal{A}$ are defined by the two conditions on the phases in the mantle ϕ_m and in the core ϕ_c

$$\left\{ \begin{array}{l} \tan\dfrac{\phi_m}{2} = \pm\sqrt{\dfrac{-\cos(2\vartheta_c)}{\cos(2\vartheta_c - 4\vartheta_m)}}, \\ \tan\dfrac{\phi_c}{2} = \pm\dfrac{\cos(2\vartheta_m)}{\sqrt{-\cos(2\vartheta_c)\cos(2\vartheta_c - 4\vartheta_m)}}. \end{array} \right. \tag{12}$$

In Fig. 2, for example, we show the contours of the transition probability at nadir angle $h = 32.4°$. The rightmost maximum corresponds to a total neutrino conversion into nonresonance region.

The total conversion can take place for $\nu_\mu \leftrightarrow \nu_s$ oscillations near the maximal vacuum mixing angle $\sin^2(2\vartheta_0) > 0.993$ and in the wide range of the vacuum mixing angles for the resonant case of $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_s$ oscillations. It can lead to a specific dependence of the nadir angle distribution for the atmospheric neutrinos. In [1] it was noted that at the maximal mixing angle $\vartheta_0 = \pi/4$, $\Delta m^2/E \cong 2 \times 10^{-4}$ eV^2/GeV and nadir angle near 30°, when the special conditions

$$\phi_m = \phi_c = \pi \tag{13}$$

are approximately satisfied, the enhancement of $\nu_\mu \leftrightarrow \nu_s$ oscillations takes place. However, the equalities (13) are not the right conditions for the maximum of the transition probability, in contrast to the opinion of the authors of ref. [1]. According to our approach these conditions correspond to the limiting case, when the absolute maxima lie on the boundary

$$\cos(2\vartheta_c - 4\vartheta_m) = 0 \tag{14}$$

of region $\mathcal{A}$. This curve defines how far from the resonance curve (9) the total neutrino conversion for the Earth-type profile can occur. The enhancement found in [1] is due to the lowest absolute maximum, which is a solution of eqs. (12), near to the maximal mixing angle (see Fig. 2).

For nonresonant matter oscillations the region, where the total neutrino conversion occurs, becomes maximal in the case of the *vacuum – matter – vacuum* profile (Fig. 3). The minimal possible vacuum mixing angle, at which the total neutrino conversion takes place, is equal to $\pi/8$, i.e. $\sin^2(2\vartheta_0) \geq 1/2$.

It has a clear physical meaning: the role of the inner layer is to prevent the rapid decrease of the transition probability, after it reaches its maximal value in the first layer (Fig. 4). Therefore, two outer layers are enough for the realization of a total neutrino conversion.

The limiting case of small mixing angles and 'optimal' conditions on the phases (10,13) is analogous to the parametric enhancement of oscillations considered in [9], where 'drift' of the transition probability to its maximal value takes place. In these cases, when oscillation amplitudes are small, many periods in a medium with periodic number density are required to reach the absolute maximum of the transition probability $P = 1$. As far as in the nonresonance region the area, where the total neutrino conversion occurs, extends more and more with each period to small mixing angles. However, the authors of refs. [9, 10] have missed all solutions for absolute maxima of the transition probability inside this area. For the Earth profile, for instance, they correspond to conditions on phases, which depend on the matter angles ϑ_m and ϑ_c (see 12). It was shown, that without the assumption about the small vacuum mixing angles or small matter effect, the transition probability can reach its absolute maximum $P = 1$ for the three-layer Earth-type medium and *even* for the two-layer case [6, 7].

So, the total conversion in a multilayer medium can occur both in the resonance and the nonresonance regions of oscillation parameters. This simple fact must be kept in mind, when the neutrino propagation in a multilayer medium with different densities like the Earth is analyzed.

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Figure captions

Figure 1a. The contours for the different values of the transition probability $P_{\mu\tau}$ 0.2, 0.4, 0.6, 0.8 at nadir angle $h = 70°$ are shown. The dark spots inside them correspond to the absolute maxima $P_{\mu\tau} = 1$, which are realized at the maximal vacuum mixing angle $\vartheta_0 = \pi/4$.

Figure 1b. The contours for the different values of the transition probability $P_{\mu s}$ 0.2, 0.4, 0.6, 0.8 at nadir angle $h = 70°$ are shown. The dark spots inside them correspond to the absolute maxima $P_{\mu s} = 1$. The resonance curve for the mantle, where the total conversion can occur, is also drawn.

Figure 2. The contours for the different values of the transition probability $P_{\mu s}$ 0.2, 0.4, 0.6, 0.8 at nadir angle $h = 32.4°$ are shown. The dark spots inside them correspond to the absolute maxima $P_{\mu s} = 1$. The region $\mathcal{A}$, where the total conversion can occur, is also drawn. For comparison the resonance curve for the mantle (dot curve) is presented.

Figure 3. The region $\mathcal{A}$ for *vacuum – matter – vacuum* profile, where the total conversion $P_{\mu s} = 1$ can occur, is plotted.

Figure 4. The evolution of the transition probability $P_{\mu s}$ in the medium *vacuum – matter – vacuum* at $\sin^2(2\vartheta_0) = 1/2$ and $\Delta m^2/E \to 0$ is presented.

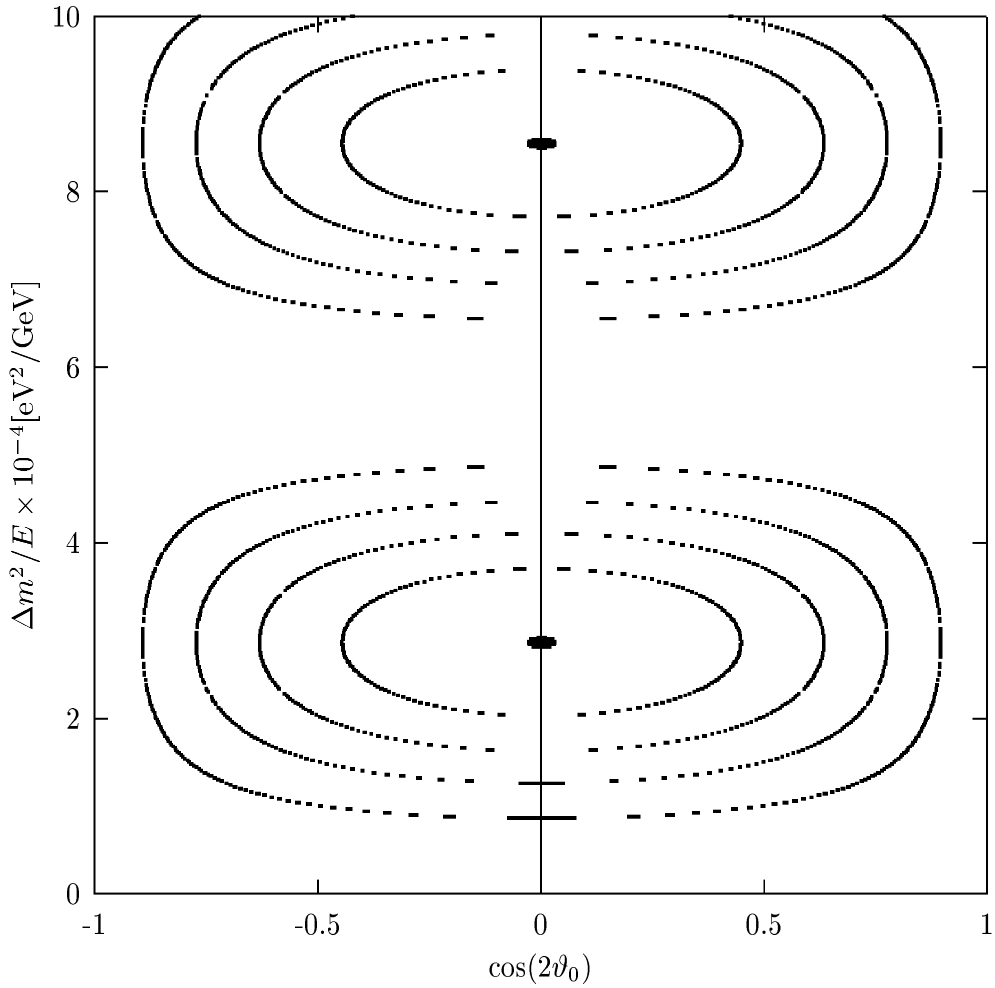
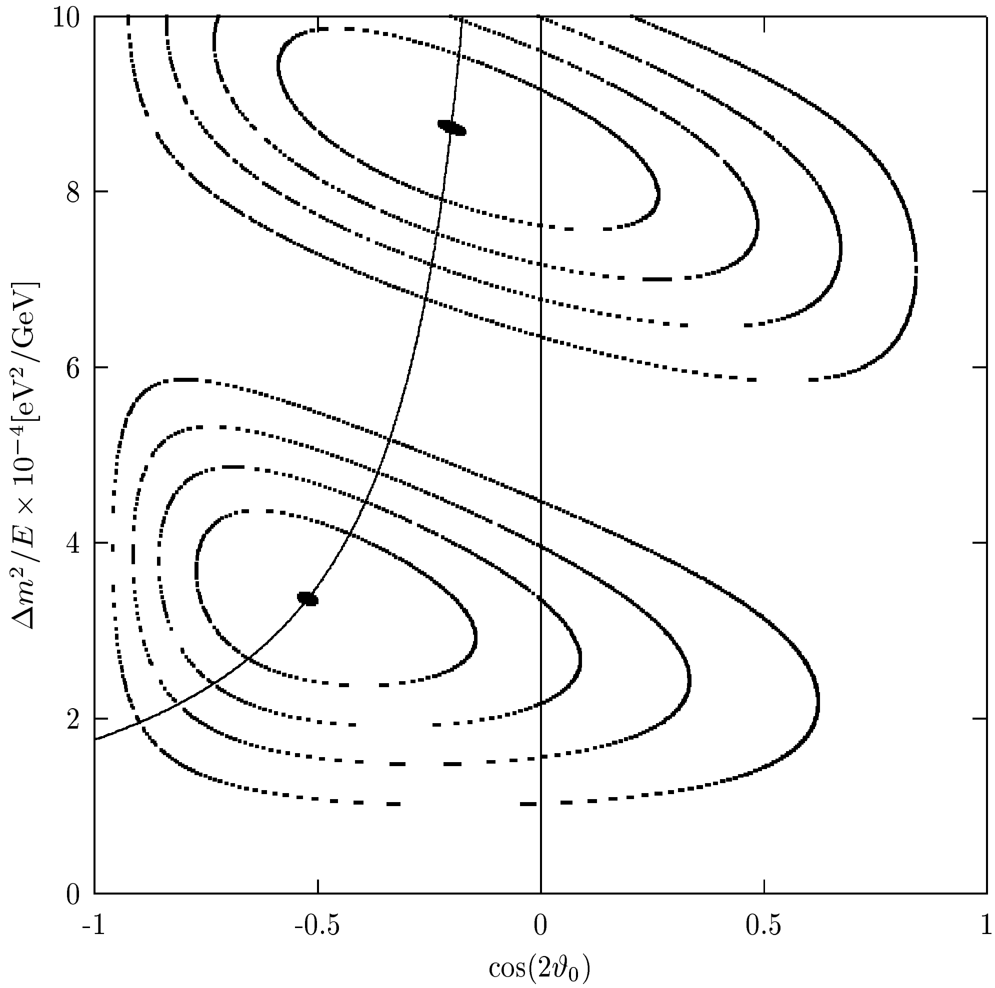


Figure 1a.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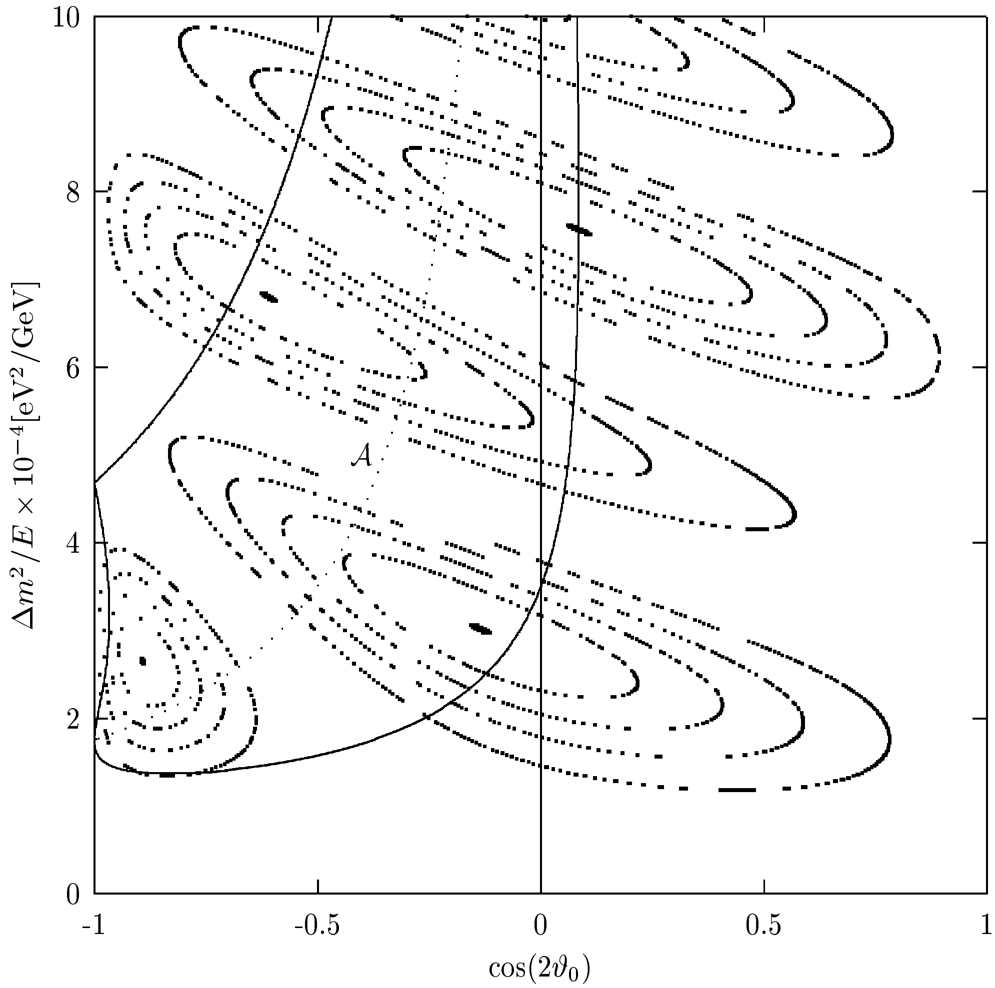
$\Delta m^2/E \times 10^{-4}[\mathrm{eV}^2/\mathrm{GeV}]$
$\cos(2\vartheta_0)$

Figure 1b.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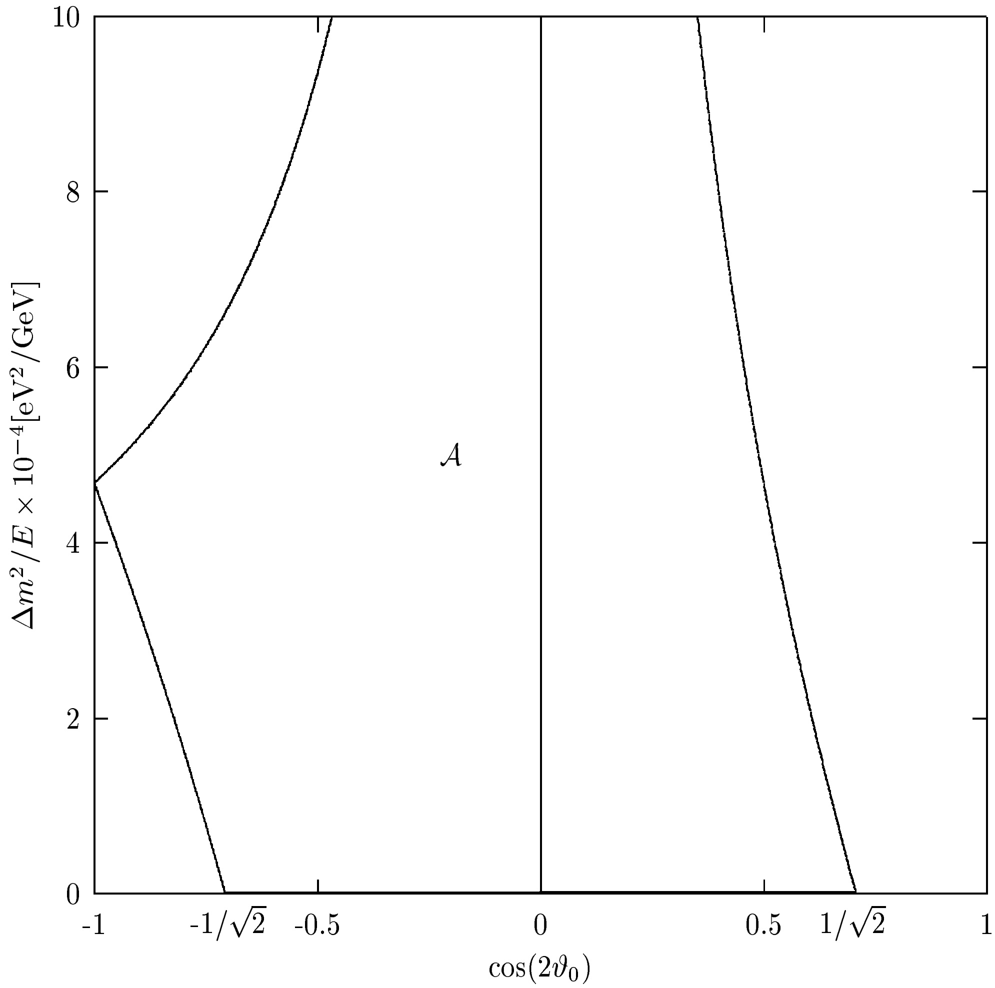
$\cos(2\vartheta_0)$
$\mathcal{A}$

Figure 2.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$1/\sqrt{2}$
1
$\cos(2\vartheta_0)$

Figure 3.

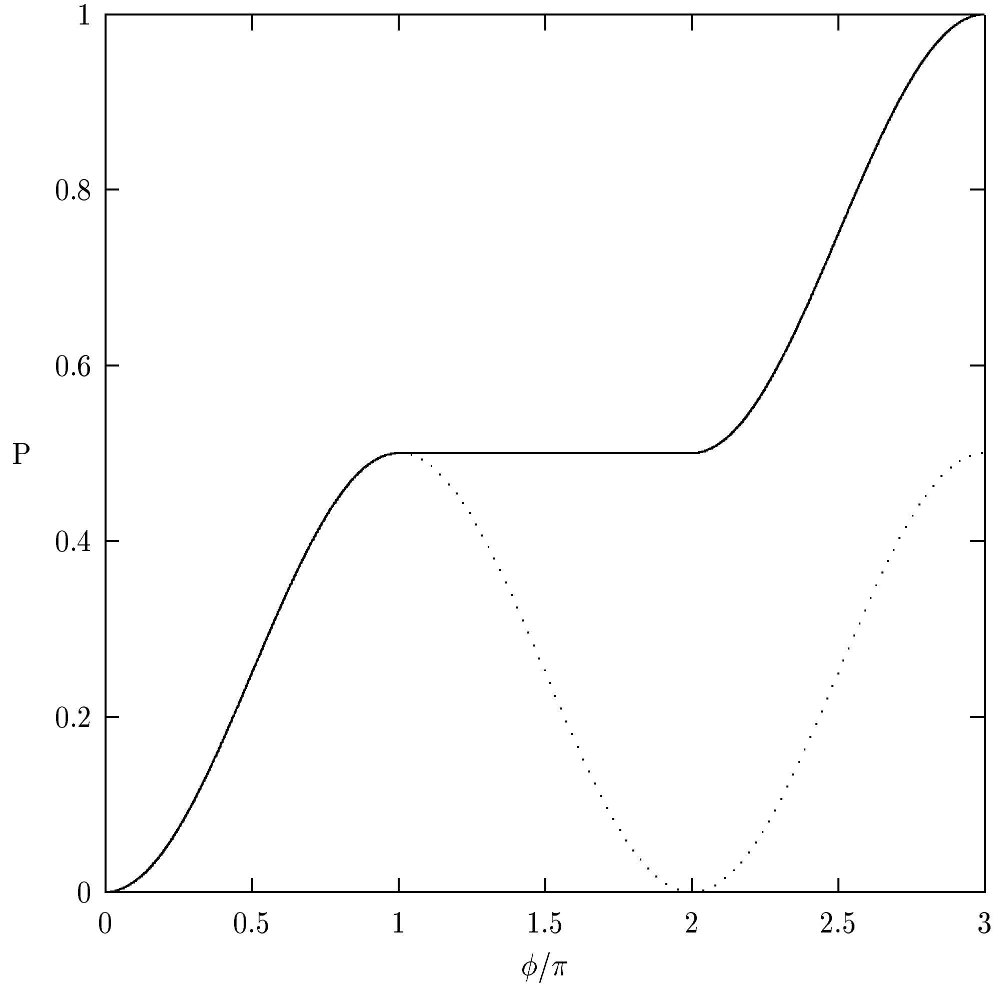

Figure 4.